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Computation of $\bar{\Lambda}$ and λ_1 with Lattice QCD

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Abstract

We pursue a new method, based on lattice QCD, for determining the quantities $\bar{\Lambda}$, λ_1, and λ_2 of heavy-quark effective theory. We combine Monte Carlo data for the meson mass spectrum with perturbative calculations of the short-distance behavior, to extract $\bar{\Lambda}$ and λ_1 from a formula from HQET. Taking into account uncertainties from fitting the mass dependence and from taking the continuum limit, we find $\bar{\Lambda} = 0.68^{+0.02}_{-0.12}$ GeV and $\lambda_1 = -(0.45 \pm 0.12)$ GeV^2 in the quenched approximation.

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In the past decade or so, heavy-quark effective theory (HQET) has become an indispensible tool for studying the physics of hadrons, such as B and D mesons, containing a single heavy quark. The main physical idea is simple: as the heavy-quark mass increases, the wave function of a "heavy-light" hadron depends less and less on the heavy-quark mass [1–3]. This is precisely as in atomic physics, where properties of hydrogen and deuterium are almost the same.

A central result from HQET is the heavy-quark expansion of a hadron's mass. Through order $1/m$, the mass M of a spin-J meson ($J = 0, 1$) is [4]

$$M = m + \bar{\Lambda} - \frac{\lambda_1}{2m} - d_J \frac{z_{\mathcal{B}}\lambda_2}{2m} + O(1/m^2), \tag{1}$$

where $d_0 = 3$ and $d_1 = -1$ tracks the spin dependence. Each term in Eq. (1) has a simple physical interpretation: m is the heavy-quark mass, the definition of which is elaborated below; $\bar{\Lambda}$ is the energy of the light quark and gluons; $-\lambda_1/2m$ is the kinetic energy of the heavy quark; and $d_J z_{\mathcal{B}}\lambda_2/2m$ is the hyperfine energy of the heavy quark's spin interacting with the chromomagnetic field inside the meson. The quantities $\bar{\Lambda}$, λ_1, and λ_2 in Eq. (1) describe the long-distance part of the bound-state problem. At long distances QCD is intrinsically nonperturbative, so it is not easy to calculate them from first principles. This should be possible with lattice gauge theory, and the aim of this Letter is to demonstrate a new method for computing $\bar{\Lambda}$, λ_1, and λ_2.

Part of the utility of HQET is that the lambdas—$\bar{\Lambda}$, λ_1, and λ_2—appear also in the heavy-quark expansions of inclusive decay spectra [5–8]. Thus, they enter into the determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $|V_{cb}|$ [9,10], and $|V_{ub}|$ [11,12]. The spin splitting $M_{B^*} - M_B$ gives a simple way to estimate λ_2, but meson masses alone are not enough to deduce $\bar{\Lambda}$ and λ_1. Moments of inclusive decay distributions [13–15] do offer a way to relate experimental data to $\bar{\Lambda}$ and λ_1, but, nevertheless, an *ab initio* QCD calculation should be of interest.

Before explaining our method for computing the lambdas, it is useful to recall how they are defined. HQET is an effective field theory, so it introduces an energy scale μ to separate long- and short-distance physics. All quantities (except d_J) on the right-hand side of Eq. (1) depend on μ and the renormalization scheme used to define it. (Meson masses remain independent of μ.) Physics from distances shorter than μ^{-1} is lumped into Wilson coefficients, such as m, $1/2m$ and $z_{\mathcal{B}}/2m$ in Eq. (1). Physics from distances longer than μ^{-1} is described by operators in the Lagrangian of HQET. The lambdas are matrix elements of these operators. When computing them one should renormalize the operators so that the lambdas are portable to the phenomenology of inclusive decays. Because those analyses compute the Wilson coefficients in perturbative QCD, it is most common to renormalize HQET in a mass-independent scheme. Then the quark mass m in Eq. (1) is the pole mass of the underlying theory, i.e., QCD. This choice of scheme obscures the μ-dependent character of m and, thus, $\bar{\Lambda}$ and λ_1, but one should still think of the pole mass as a special choice of perturbative short-distance mass. The scheme is easily portable, because the pole mass is infrared finite and gauge independent at every order in perturbative QCD [16], and the relation between the pole and $\overline{\text{MS}}$ masses in QCD is known through order α_s^3 [17].

Another property of the lambdas is that they are independent of the heavy-quark mass (if, as we do, one distinguishes μ from m). HQET starts with the infinite-mass limit, or static effective theory [2,18,19]. The eigenstates of this theory are independent of m. One can then develop the expansion in $1/m$ of the underlying theory (QCD) around the infinite-mass limit, so that matrix elements are taken in the infinite-mass eigenstates [20,21,4].

Our lattice method retains the logic and structure of the usual application of HQET. Lattice gauge theory with Wilson fermions has a stable heavy-quark limit [22], in which the Isgur-Wise heavy-quark symmetries are prominent. Indeed, the static limit is the same as for continuum QCD. Consequently, one may apply HQET directly to lattice gauge theory, to separate long- from short-distance physics [23]. The key difference is that there are now *two* short distances, $1/m$ and the lattice spacing a. That does not run afoul of the assumptions of HQET; it means merely that the short-distance coefficients must be modified to depend on a as well as m. Then one may use HQET to develop heavy-quark expansions for lattice observables. The expansion for the rest mass M_1 of a spin-J meson is [23]

$$M_1 = m_1 + \bar{\Lambda}_{\text{lat}} - \frac{\lambda_{1\text{lat}}}{2m_2} - d_J \frac{\lambda_{2\text{lat}}}{2m_{\mathcal{B}}} + O(1/m^2), \tag{2}$$

where m_1, $1/2m_2$, and $1/2m_{\mathcal{B}} = z_{\mathcal{B}\text{lat}}/2m_2$ are the modified short-distance coefficients. The rest mass and kinetic mass M_2 are defined through the energy

$$E(\boldsymbol{p}) = M_1 + \frac{\boldsymbol{p}^2}{2M_2} + \cdots \tag{3}$$

of a state with small momentum $\boldsymbol{p}$. Because the lattice breaks Lorentz invariance, M_2 need not equal M_1, except asymptotically as $Ma \to 0$. For quarks m_1 and m_2 are defined similarly in matching calculations.

As $ma \to 0$ lattice QCD becomes continuum QCD, so then $m_{1,2} \to m$ and $z_{\mathcal{B}\text{lat}} \to z_{\mathcal{B}}$. Owing to limitations in computer resources there are, however, no lattice data available with $ma \ll 1$ and $m \gg \Lambda_{\text{QCD}}$. The advantage of Eq. (2) is that it holds for general ma, as long as $m_{2,\mathcal{B}} \gg \Lambda_{\text{QCD}}$. One may, therefore, apply Eq. (2) to published data for M_1.

Like their continuum-QCD counterparts, the quantities $\bar{\Lambda}_{\text{lat}}$, $\lambda_{1\text{lat}}$, and $\lambda_{2\text{lat}}$ do not depend on the heavy-quark mass. They are labeled with the subscript "lat" because the gluons and light quarks are also on the lattice. Their lattice-spacing dependence can be separated from continuum QCD with Symanzik's formalism [24], which implies, for example,

$$\bar{\Lambda}_{\mathrm{lat}} = \bar{\Lambda} + aC_1\mathcal{M}_1 + a^2C_2\mathcal{M}_2 + \cdots, \tag{4}$$

where the C_i represent short-distance coefficients and the $\mathcal{M}_i$ long-distance matrix elements in Symanzik's effective Lagrangian. Equation (4) is a good guide for extrapolating $a \to 0$ as soon as $\Lambda_{\mathrm{QCD}}a \ll 1$. Because lattice-spacing effects of the heavy quark are isolated in Eq. (2), it does not matter if ma is not small.

Our method is to take Monte Carlo data for M_1 over a wide range of heavy quark masses, combine them with separate calculations of the short-distance coefficients, and perform fits to Eq. (2). This is very simple for $\lambda_{2\mathrm{lat}}$:

$$\tfrac{1}{2}m_{\mathcal{B}}(M_{1B^*} - M_{1B}) = \lambda_{2\mathrm{lat}}, \tag{5}$$

with quark masses $m_2 \gg \Lambda_{\mathrm{QCD}}$ and with fixed μ (and a). For $\bar{\Lambda}_{\mathrm{lat}}$ and $\lambda_{1\mathrm{lat}}$ we consider the spin-averaged rest mass $\bar{M}_1 := \frac{1}{4}(3M_{1B^*} + M_{1B})$. Then $\lambda_{2\mathrm{lat}}$ drops out, and Eq. (2) becomes

$$\bar{M}_1 - m_1 = \bar{\Lambda}_{\mathrm{lat}} - \frac{\lambda_{1\mathrm{lat}}}{2m_2}. \tag{6}$$

Equation (6) is the crux of our analysis: we plot the combination on the left-hand side against $(2m_2)^{-1}$, and a fit to the mass dependence yields $\bar{\Lambda}_{\mathrm{lat}}$ and $-\lambda_{1\mathrm{lat}}$. We repeat this procedure for several lattice spacings to take the continuum limit, guided by Eq. (4).

To carry out the analysis one must calculate M_1, for vector and pseudoscalar mesons, and the short-distance coefficients m_1, m_2 and $m_{\mathcal{B}}$. For the coefficients we shall use perturbative QCD. In lattice gauge theory

$$m_X = m_X^{[0]} + \sum_{l=1}^{\infty} g_0^{2l}(1/a)m_X^{[l]}, \tag{7}$$

where $g_0^2(1/a)$ is the bare coupling for a lattice with spacing a. For m_1 and m_2, Ref. [25] derived formulas to relate the higher-order terms to the self energy and gave the one-loop terms $m_X^{[1]}$ for the lattice action used below. For $m_{\mathcal{B}}$ only the tree-level term $m_{\mathcal{B}}^{[0]}$ is known, so, for now, we cannot obtain a meaningful result for $\lambda_{2\mathrm{lat}}$.

It is well-known that perturbation theory in $g_0^2(1/a)$ converges poorly. Therefore, we re-express Eq. (7) in a renormalized coupling, chosen with the Brodsky-Lepage-Mackenzie (BLM) prescription [26]. For a coupling in scheme S, we denote the BLM expansion parameter $g_S^2(q_S^*)$. The BLM scale q_S^* is given by

$$\log q_S^* = -\tfrac{1}{2}b_S^{(1)} + \frac{\int d^4k\, \log k\, f(k)}{\int d^4k\, f(k)}, \tag{8}$$

where k is the gluon momentum, and $f(k)$ is the integrand of the quantity of interest, e.g., $\int d^4k\, f(k) = m_1$. The constant $b_S^{(1)}$ is the β_0-dependent part of the one-loop conversion from the arbitary scheme S to the "V scheme", namely

$$\frac{(4\pi)^2}{g_S^2(q)} = \frac{(4\pi)^2}{g_V^2(q)} + \beta_0 b_S^{(1)} + b_S^{(0)} + O(g^2), \tag{9}$$

where for n_f light quarks $\beta_0 = 11 - 2n_f/3$, and $b_S^{(0)}$ is independent of n_f. The V-scheme coupling $g_V^2(q)$ is defined so that the Fourier transform of the heavy-quark potential reads $V(q) = -C_F g_V^2(q)/q^2$. Equation (8) shows that the definitions of q^* in Refs. [26] and [27] are equivalent in the V scheme.[1]

The purpose of the logarithmically weighted integral in Eq. (8) is to sum up into g_S^2 higher-order terms of order $g^2(\beta_0 g^2)^{l-1}$, $l > 1$, which with a foolish choice of scale would be large. The purpose of the constant is to make $g_S^2(q_S^*)$ independent of S, apart from contributions of order $g^4(\beta_0 g^2)^{l-2}$. This is an advantage in matching calculations: it makes little numerical difference whether one re-expands Eq. (7) in $g_0^2(q_0^*)$ or $g_V^2(q_V^*)$.

In practice, we use $g_V^2(q_V^*)$, computed from the 1×1 Wilson loop and $g_V^2(3.40/a)$ as in Ref. [27]. For m_1 the BLM scale $q_V^* = q_1^*$ is now available [29]. Most of the loop correction to m_2 can be attributed to m_1, leaving an additional renormalization factor Z_{m_2} [25]. The one-loop term is small [25], but the BLM scale q_2^* is not yet available. So, for Z_{m_2} we simply use $q_2^* = q_1^* \pm 20\%$, fully correlated, and tolerate an extra uncertainty.

For lattice meson masses M_1 we select numerical data from recent work on heavy-light pseudoscalar and vector mesons [30–32]. The data are tabulated in Table I. For uniformity, the value in physical units of the lattice spacing a is defined according to the suggestion of Sommer [34]. (It gives the same numerical result as the 1P-1S splitting of charmonium.) The lattice spacing varies by a factor of nearly 3, allowing us the take the continuum limit as guided by Eq. (4). All data sets are in the quenched approximation, which omits the back-reaction of light quarks on the gluons and partly compensates the omission by implicit shifts in the bare couplings. Light quarks have the Sheikholeslami-Wohlert action [33], to minimize discretization effects on the light quark. In most data sets, the (physical) quark mass spans a range from near charm to slightly above beauty, allowing us to examine the mass dependence of Eq. (6).

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[1] For convenience, we list some of the $b_S^{(i)}$ here. In the V scheme $b_V^{(1)} = b_V^{(0)} = 0$, by definition; in the $\overline{\mathrm{MS}}$ scheme $b_{\overline{\mathrm{MS}}}^{(1)} = 5/3$, $b_{\overline{\mathrm{MS}}}^{(0)} = -8$; for the bare gauge coupling [28] $b_0^{(1)} = b_{\overline{\mathrm{MS}}}^{(1)} - 6\pi K_1(1) = 9.12637$, $b_0^{(0)} = b_{\overline{\mathrm{MS}}}^{(0)} + 2\pi[2d_{10} + 33K_1(1)] = -16.1213$.

TABLE I. Numerical and perturbative results used in this paper. The first column cites the source of the numerical data. The second column includes the plotting symbol used in all figures. Statistical errors are given for a^{-1} and $\bar{M}_1$; systematic (perturbative) errors for m_2. Perturbative results are from input data to the numerical calculations and Refs. [25,29].

Ref.	a^{-1} (GeV)	$\bar{M}_1 a$	$m_1 a$	$m_2 a$	$q_1^* a$
[30]	3.35(2)	1.731(3)	1.560	2.761(54)	0.91
	octagons	1.301(3)	1.122	1.567(21)	0.90
		0.946(2)	0.761	0.903(7)	0.85
		0.789(2)	0.602	0.674(4)	0.80
		0.667(2)	0.477	0.514(2)	0.75
		0.589(2)	0.398	0.420(1)	0.70
		0.523(2)	0.331	0.343(1)	0.65
	2.50(2)	2.147(4)	1.943	4.216(132)	0.90
	squares	1.611(3)	1.399	2.217(49)	0.91
		1.183(3)	0.961	1.229(17)	0.88
		0.978(3)	0.750	0.880(9)	0.84
		0.845(3)	0.613	0.686(5)	0.80
		0.749(2)	0.514	0.558(3)	0.76
		0.676(2)	0.438	0.466(2)	0.71
[31]	2.50(2)	2.557(7)	2.364	6.722(234)	0.88
	crosses	1.403(14)	1.200	1.716(29)	0.90
		0.726(10)	0.504	0.545(3)	0.75
	1.77(1)	2.665(6)	2.422	6.773(373)	0.88
	diamonds	1.663(4)	1.402	2.163(66)	0.91
		0.964(4)	0.677	0.770(9)	0.81
		0.876(4)	0.582	0.642(6)	0.77
	1.16(1)	2.829(6)	2.535	6.735(806)	0.88
	fancy	2.345(6)	2.037	4.067(381)	0.90
	squares	1.935(6)	1.612	2.599(179)	0.91
		1.489(5)	1.139	1.498(63)	0.89
		1.274(5)	0.917	1.112(34)	0.85
[32]	2.90(2)	0.958(7)	0.748	0.883(8)	0.84
	fancy	0.849(6)	0.636	0.719(5)	0.81
	diamonds	0.762(6)	0.548	0.602(3)	0.78
		0.670(5)	0.454	0.485(2)	0.73

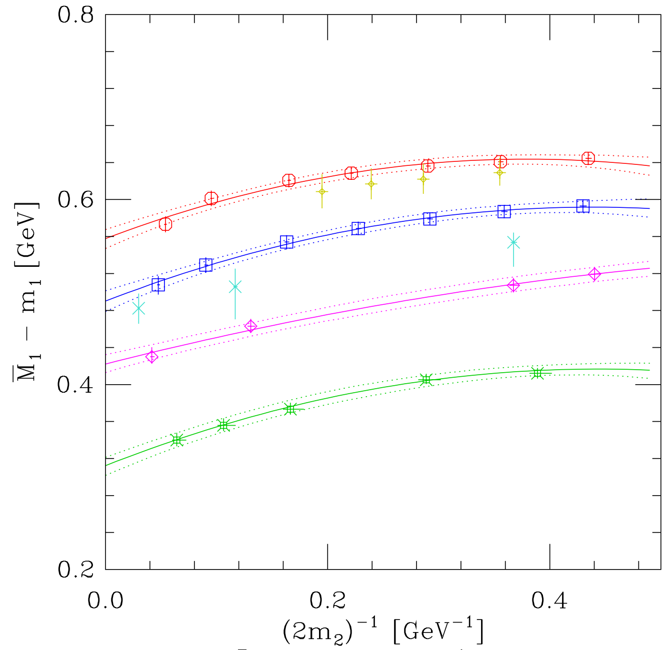

FIG. 1. Plot of $\bar{M}_1 - m_1$ vs. $(2m_2)^{-1}$. The key for the plotting symbols is given in Table I. For clarity the error envelopes for the crosses and fancy diamonds are not shown.

Figure 1 plots $\bar{M}_1 - m_1$ vs. $(2m_2)^{-1}$. The vertical error bars reflect statistical uncertainties only, and the horizontal error bars reflect these and the variation in q_2^*. There is noticeable curvature, which is not surprising because the data reach masses below the charmed quark mass. We handle the curvature in two ways. First, we fit linearly the subset of data with $m_2 \geq 2.5$ GeV. Second, we extend Eq. (6) to order $1/m^2$ [23]:

$$\bar{M}_1 - m_1 = \bar{\Lambda}_{\rm lat} - \frac{\lambda_{1\rm lat}}{2m_2} + \frac{\rho_{1\rm lat}}{4m_D^2} - \frac{\mathcal{T}_{1\rm lat} + \mathcal{T}_{3\rm lat}}{(2m_2)^2}, \quad (10)$$

where $1/4m_D^2$ is the short-distance coefficient of the Darwin term, and $\rho_{1\rm lat}$, $\mathcal{T}_{1\rm lat}$, and $\mathcal{T}_{3\rm lat}$ are matrix elements of higher-dimension terms [35–37], with the notation of Ref. [37], for gluons and light quarks on a lattice. The $1/m^2$ terms are important for smaller masses, where $m_D \approx m_2$ within the precision available. Thus, only one unknown is needed to model the curvature.

We take the second method as our standard and use the first for comparison. The solid curves in Fig. 1 are the best fit to Eq. (10). We use the bootstrap method to propagate the underlying uncertainties through the fit. In this way we account fully (partially) for correlations in the data from Ref. [31] (Refs. [30,32]). The dotted lines show the error envelopes of the fits; they hug the best fit in the region of interpolation and flare out in the region of extrapolation.

As expected, $\bar{\Lambda}_{\rm lat}$ and $\lambda_{1\rm lat}$ depend on the lattice spacing a. For the data sets used, the coefficient C_1 in Eq. (4) is of order α_s and the coefficient C_2 is of order 1. Asymptotically, the former dominates, so we fit $\bar{\Lambda}_{\rm lat}$ linearly in a

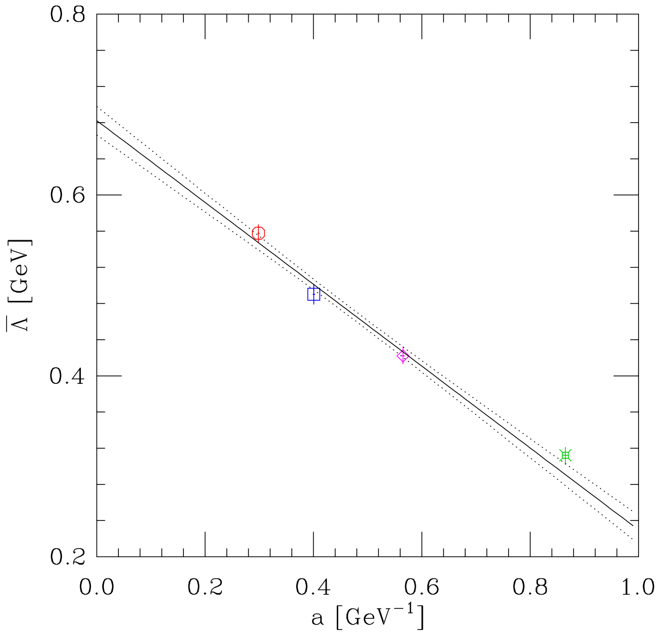

FIG. 2. Continuum limit of $\bar{\Lambda}$.

to take the continuum limit. The slope, $C_1\mathcal{M}_1$, is somewhat large for a quantity of order $\alpha_s\Lambda^2_{\rm QCD}$, so we also consider fits linear in a^2. The χ^2/dof is smaller for the fit linear in a, so we take it for our central value and take the other to quote a systematic error. In future work, one should tune the light quark action so that C_1 is of order a [38]; then the extrapolation Ansatz would be unambiguous.

Figure 2 plots $\bar{\Lambda}_{\rm lat}$ vs. a. The error bars are from the bootstrap of the fit described above. From now on we discard the data sets denoted in Fig. 1 by crosses and fancy diamonds. Their error bars are very large: the crosses have too few points and the mass range of the fancy diamonds is too small. $\bar{\Lambda}_{\rm lat}$ exhibits significant dependence on a; in this case, it would have been misleading to determine $\bar{\Lambda}$ with data at only one lattice spacing.

Figure 3 plots $\lambda_{1\rm lat}$ vs. a. The error bars are again from the bootstrap of the mass fit. In this case, lattice spacing effects are smaller than other uncertainties, and it does not matter whether we take the continuum limit with a fit to a or to a^2.

The results exhibit a strong correlation in the $\bar{\Lambda}$-λ_1 plane, as shown in Fig. 4. The points show the scatter from the bootstrap method. The ellipses surround 68% of the points. Dark grey (red) points show the standard analysis, with fits quadratic in $1/2m_2$ and linear in a. Light grey (blue) points show the analysis with continuum extrapolation linear in a^2, yielding smaller $\bar{\Lambda}$. The results from four different Ansätze for fitting are tabulated in Table II.

Clearly the choice of lattice-spacing extrapolation dominates the uncertainties of Monte Carlo statistics and q_2^*, which are propagated carefully through the fits.

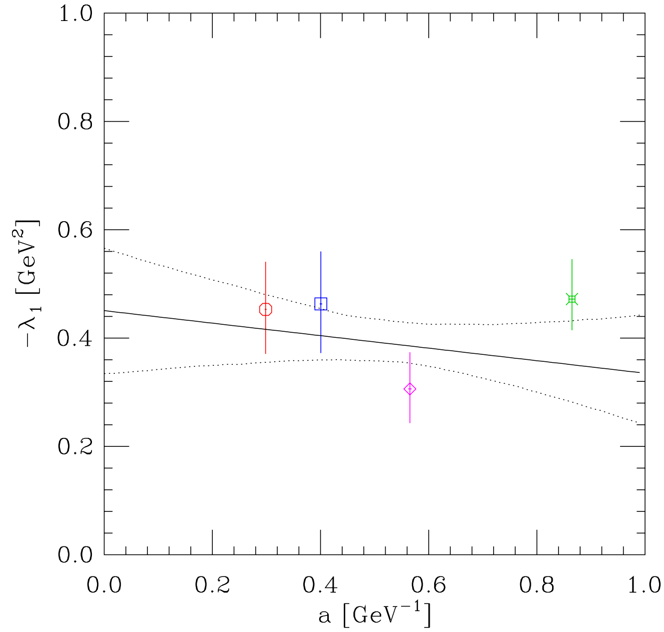

FIG. 3. Continuum limit of λ_1.

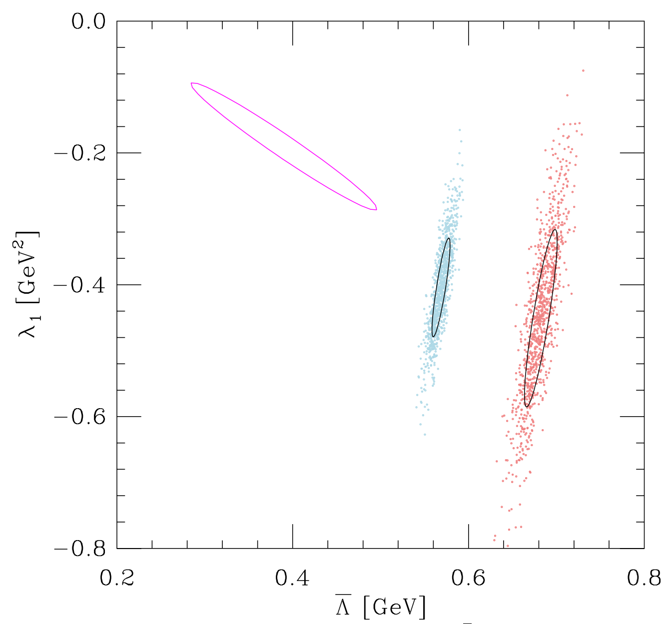

FIG. 4. Correlation of our results for $\bar{\Lambda}$ and λ_1 from two analyses of the continuum limit. Dark grey (red) points are the standard analysis, quadratic in $1/2m_2$ and linear in a. Light grey (blue) points are quadratic in $1/2m_2$ and linear in a^2, yielding smaller $\bar{\Lambda}$. The ellipse in the upper left is the result of Ref. [13].

TABLE II. Numerical results for four different fit Ansätze. The column labeled ρ gives the normalized coefficient of correlation.

fit	$\bar{\Lambda}$ (GeV)	$-\lambda_1$ (GeV^2)	ρ
Eq. (10), a	0.68 ± 0.02	0.45 ± 0.12	0.869
Eq. (6), a	$0.67^{+0.01}_{-0.02}$	0.44 ± 0.11	0.852
Eq. (10), a^2	0.57 ± 0.01	0.40 ± 0.07	0.860
Eq. (6), a^2	0.57 ± 0.01	0.41 ± 0.08	0.871

To account for this, we take central value for $\bar{\Lambda}$ from the standard analysis, but we extend the error bar to encompass the full range suggested by the a^2 fit. On the other hand, the standard fit gives an error bar for $-\lambda_1$ that covers the range of the other fits, so we simply use it. With these considerations we find,

$$\bar{\Lambda} = 0.68^{+0.02}_{-0.12}\ \mathrm{GeV}, \tag{11}$$

$$\lambda_1 = -(0.45 \pm 0.12)\ \mathrm{GeV}^2. \tag{12}$$

The standard fit also yields an estimate of dimension-three combination $\mathcal{T}_1 + \mathcal{T}_3 - \rho_1 = 0.51 \pm 0.22\ \mathrm{GeV}^3$.

The orientation of the ellipses from our method is roughly orthogonal to that found from moments of the lepton energy spectrum [13,14] or the hadronic invariant mass spectrum [15] of inclusive B decay. For illustration, the former is shown in Fig. 4 as well.

There are two uncertainties that we cannot yet address fully. One is the effect of finite volume on M_1. Studies of the volume dependence of heavy-light systems [39,40] suggest that finite-volume effects are negligible compared to our other uncertainties. A more serious uncertainty arises because the numerical data were generated in the quenched approximation. One may expect that the shift in $\bar{\Lambda}$ owing to quenching is small, for the same reason that the shift in the heavy-quark mass is small [41]. A qualitative way of estimating the effect of quenching is to check other, similar observables. With Sommer's definition of a one finds discrepancies in m_ρ of around -10 percent, suggesting that $\bar{\Lambda}$ could be 10 percent smaller, and λ_1 20 percent larger, than quoted here.

We do not quote an uncertainty from the perturbative calculation of the short-distance effects. Because HQET, as customarily applied, is defined with a perturbative renormalization scheme, any application suffers from such uncertainties. Our results for $\bar{\Lambda}$ and λ_1 can be used consistently with the pole mass in next-to-leading order, BLM-improved phenomenology. In such an application a single uncertainty from truncating perturbative QCD should be quoted. Indeed, because the pole mass has large higher-order contributions, so does $\bar{\Lambda}$, but in a physical application the large terms cancel. If next-to-next-to-leading accuracy is required, then the analysis presented here must be repeated with (as yet uncalculated) two-loop short-distance coefficients.

Our central value for $\bar{\Lambda}$ is somewhat larger than those from QCD sum rules [42], but taking the uncertainties into account, there is no inconsistency. Our result for λ_1 agrees with some sum-rule estimates, but not others [43]. It is not clear what to make of the discrepancies in sum rules. Our uncertainties are reducible, and below we identify ways to improve the numerical data that go into our analysis.

In the past, there have been attempts to calculate the lambdas in a discretization of the infinite-mass limit [44]. This method faces two difficulties. First, it yields the lambdas in a lattice renormalization scheme, and the results must be converted to the continuum schemes in common use. The conversion must deal with power-law divergences [45]. Second, it identifies the HQET separation scale μ with the ultraviolet cutoff π/a of the gluons, so it is hard to take the continuum limit. Our method circumvents these obstacles by formulating HQET as an effective field theory to describe sets of (lattice) data. In this way HQET obtains its own scale μ and the second problem does not arise. The first problem arises from taking $m \to \infty$ with a fixed. Our method sidesteps it by fitting the mass dependence in the regime $m_1 a \lesssim 2$, and, since $m \gg \Lambda_{\mathrm{QCD}}$, HQET identifies the fit parameters with $\bar{\Lambda}_{\mathrm{lat}}$, $\lambda_{1\mathrm{lat}}$, and $\lambda_{2\mathrm{lat}}$.

In this paper, we have presented a new way to determine $\bar{\Lambda}$, λ_1, and λ_2. Using numerical data in the literature, we have shown that it is feasible to carry out the necessary fit in quark mass and extrapolation in lattice spacing to obtain encouraging results. Systematic uncertainties in the mass extrapolation might be improved using the hopping-parameter expansion [46], to create a continuous range of heavy-quark mass. With small enough statistical errors and a wide enough range of data, it might be possible also to extract the dimension-three quantities ρ_i and $\mathcal{T}_i$, although that task requires the calculation of several additional short-distance coefficients. Similarly, systematic uncertainties in the lattice-spacing extrapolation could be improved by adjusting the light quarks' action so that C_1 in Eq. (4) is rendered of order a [38]. Finally, our methods could be applied to full QCD, once such data sets have been generated, to obtain truly *ab initio* results.

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